

Meta-stable A_n quiver gauge theories

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ABSTRACT: We study metastable dynamical breaking of supersymmetry in A_n quiver gauge theories. We present a general analysis and criteria for the perturbative existence of metastable vacua in quivers of any length. Different mechanisms of gauge mediation can be realized.

KEYWORDS: Supersymmetry Breaking, Supersymmetry and Duality, Supersymmetry Phenomenology.

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1. Introduction

The existence of long living metastable vacua [1, 2] seems by now a rather generic phenomenon in large classes of supersymmetric gauge theories [3–5]. It provides an attractive way for dynamical breaking of supersymmetry and the interest in these theories has been enhanced by the possibilities of their embedding in supergravity and string theory [6] and of their use [7–10] in gauge mediation mechanisms [11].

Metastability is a low energy phenomenon for UV free theories and in general the key ingredient which makes a perturbative analysis possible is Seiberg duality to IR free theories described in terms of macroscopic fields [12].

An interesting set of theories in which to study metastability à la ISS [1] is the ADE class of quiver gauge theories [13, 14].

These theories can be derived in type *IIB* string theory from *D5*-branes partially wrapping 2-cycles of non compact Calabi-Yau threefolds. These manifolds are ADE-fold geometries fibered over a plane, and the 2-cycles are blown up S_i^2 in one to one correspondence with the simple roots of ADE.

In this paper we investigate metastability in A_n $\mathcal{N} = 2$ (non affine) quiver gauge theories deformed to $\mathcal{N} = 1$ by superpotential terms in the adjoint fields. In the presence of many gauge groups we have, in principle, a large number of dualization choices.

In [3, 8, 9] A_2, A_3, A_4 quivers have been studied dualizing only one node in the quiver, where dynamical supersymmetry breaking occurs.

Here we consider A_n theories with arbitrary n , where several Seiberg dualities take place. In particular we will explore theories obtained by dualizing alternate nodes. This leads to a low energy description in terms of only magnetic fields.

In the duality process the dualized groups are treated as genuine gauge groups whereas the other ones have to be weakly coupled at low energy, so that they act as flavour groups i.e. global symmetries. The procedure depends on the interplay of the RG flows of the dualized and of the non dualized gauge groups and is governed by the associated beta-functions. This translates into inequalities among the ranks of the gauge groups and in hierarchies among the strong coupling scales.

The paper is organized as follows. In section 2 we describe the $\mathcal{N} = 2$ quiver gauge theories, explicitly broken to $\mathcal{N} = 1$ by superpotential terms. After the integration of the massive adjoint fields, we give the general form of the superpotential. In section 3 we investigate Seiberg duality on the alternate nodes of the quiver. The general theory obtained with this procedure on an A_n is expressed in terms of only magnetic fields. In section 4 we consider the simplest case, i.e. A_3 quiver, showing that it possesses long living metastable vacua à la ISS. The analysis is done neglecting the gauge contributions of the odd nodes, which are treated as flavour symmetries. This last approximation is justified in section 5, where an analysis of the running of the couplings has been performed. The general result, metastability in an A_n quiver theory, is explained in section 6, giving an explicit example. In section 7 we comment on the possible ways of enforcing gauge mediation of supersymmetry breaking. Appendix A explains how to find the metastable vacua upon changing the masses of the quarks in the electric description. Appendix B provides details in the analysis on the running of the gauge couplings of section 5. Appendix C adds to section 6, giving all the possible choices of A_5 which show metastable vacua.

2. A_n quiver gauge theories with massive adjoint fields

We consider a $\mathcal{N} = 2$ (non affine) A_n quiver gauge theory, deformed to $\mathcal{N} = 1$ by superpotential terms in the adjoint fields. The theory is associated with a Dynkin diagram where each node is a $U(N_i)$ gauge group (figure 1).

The arrows connecting two nodes represent fields $Q_{i,i+1}, Q_{i+1,i}$ in the fundamental of the incoming node and anti fundamental of the out-coming node. The adjoint fields X_i refer to the i -th gauge group.

The gauge group of the whole theory is the product $\prod_{i=1}^n U(N_i)$. We call Λ_i the strong coupling scale of each gauge group.

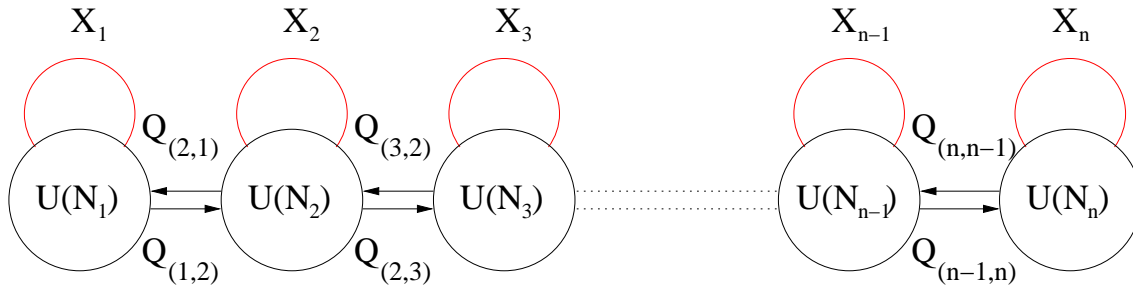


Figure 1: A_n quiver.

The $\mathcal{N} = 1$ superpotential is

$$W = \sum_{i=1}^n W_i(X_i) + \sum_{i,j} s_{i,j} (Q_{i,j})_{\alpha}^{\beta} (X_j)_{\beta}^{\gamma} (Q_{j,i})_{\gamma}^{\alpha} \quad (2.1)$$

where $s_{i,j}$ is an antisymmetric matrix, with $|s_{i,j}| = 1$. The Latin labels run on the different nodes of the A_n quivers, the Greek labels run on the ranks of the groups of each site. In the case of A_n theories the only non zero terms are $s_{i,i+1}$ and $s_{i,i-1}$. The superpotentials for the adjoint fields $W_i(X_i)$ break supersymmetry to $\mathcal{N} = 1$.

We choose these superpotentials to be

$$W_i(X_i) = \lambda_i \text{Tr} X_i + \frac{m_i}{2} \text{Tr} X_i^2 \quad (2.2)$$

As a consequence the adjoint fields are all massive. We consider the limit where the adjoint fields are so heavy that they can be integrated out, and we study the theory below the scale of their masses.

Integrating out these fields we obtain the effective superpotential describing the A_n theory (traces on the gauge groups are always implied).

$$W = \sum_{i=1}^{n-1} \left(\left(\frac{\lambda_{i+1}}{m_{i+1}} - \frac{\lambda_i}{m_i} \right) Q_{i,i+1} Q_{i+1,i} - \frac{1}{2} \left(\frac{1}{m_i} + \frac{1}{m_{i+1}} \right) (Q_{i,i+1} Q_{i+1,i})^2 \right) + \sum_{i=2}^{n-1} \frac{1}{m_i} Q_{i-1,i} Q_{i,i+1} Q_{i+1,i} Q_{i,i-1} \quad (2.3)$$

A final important remark is that for the A_n theories the D -term equations of motion can be decoupled and simultaneously diagonalized [15].

3. Seiberg duality on the even nodes

We investigate the low energy dynamics of the gauge groups of the Dynkin diagram, governed by the ranks and by the hierarchy between the strong coupling scales of each node. We work in the regime where the even nodes develop strong dynamics and have to be Seiberg dualized.

	$U(N_{2i-1})$	$U(\widetilde{N}_{2i})$	$U(N_{2i+1})$
$M_{2i+1,2i-1}$	N_{2i-1}	1	\widetilde{N}_{2i+1}
$M_{2i+1,2i+1}$	1	1	Bifund.
$M_{2i-1,2i-1}$	Bifund.	1	1
$M_{2i-1,2i+1}$	\widetilde{N}_{2i-1}	1	N_{2i+1}
$q_{2i-1,2i}$	N_{2i-1}	\widetilde{N}_{2i}	1
$q_{2i,2i-1}$	\widetilde{N}_{2i-1}	\widetilde{N}_{2i}	1
$q_{2i,2i+1}$	1	\widetilde{N}_{2i}	\widetilde{N}_{2i+1}
$q_{2i+1,2i}$	1	\widetilde{N}_{2i}	N_{2i+1}

Table 1: Fields in the magnetic description.

We set all the strong coupling scales of the even nodes to be equal $\Lambda_{2i} \equiv \Lambda_G$ and we require the odd nodes to be less coupled at this scale. We impose the following window for the ranks of the nodes

$$N_{2i} + 1 \leq N_{2i-1} + N_{2i+1} < \frac{3}{2}N_{2i} \quad i = 1, \dots, \frac{n-1}{2} \quad (3.1)$$

We take n odd, the even case can be included setting to zero one of the ranks of the extremal nodes.

Along the flow toward the IR, we have to change the description at the scale Λ_G performing Seiberg duality on the even nodes. The even nodes are treated as gauge groups, whereas the odd nodes are treated as flavours. We will discuss the consistency of this description in section 5.

It is convenient to list the elementary fields of the dualized theory, i.e. the electric gauge singlets and the new magnetic quarks (table 1).

The mesons are proportional to the original electric variables: $M_{2i+k,2i+j} \sim Q_{2i+k,2i} Q_{2i,2i+j}$. The even magnetic groups have ranks $\widetilde{N}_{2i} = N_{2i+1} + N_{2i-1} - N_{2i}$. The superpotential in the new magnetic variables results

$$W = hM_{2i+k,2i+j}^{(2i)} q_{2i+j,2i} q_{2i,2i+k} + h\mu_{2i+k,(2i)}^2 M_{2i+k,2i+k}^{(2i)} + \quad (3.2)$$

$$+ hmM_{2i+1,2i+1}^{(2i)} M_{2i+1,2i+1}^{(2i+2)} + hm \left(M_{2i+k,2i+k}^{(2i)} \right)^2 + hmM_{2i-1,2i+1}^{(2i)} M_{2i+1,2i-1}^{(2i)}$$

where the index i runs from 1 to $\frac{n-1}{2}$, and k and j are $+1$ or -1 . The upper index $(2i)$ of the mesons indicates which site the meson refers to: it is necessary because some mesons have the same flavor indexes, but they are summed on different gauge groups, so they have to be labeled differently. We denote with hm_i the meson masses, related to the quartic terms in the electric superpotential, and with $h\mu_i^2$ the coefficients of the linear deformations, corresponding to the masses of the quarks in the electric description. In (3.2) we wrote a single coupling hm , for all the different mesons, considering all their masses of the same order.

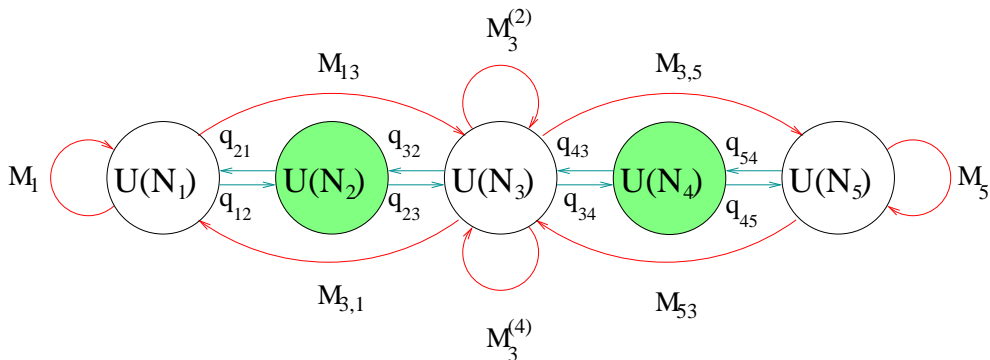


Figure 2: A_5 dual quiver.

The b coefficients of the beta functions before dualization are

$$b_i = 3N_i - N_{i-1} - N_{i+1} \quad i = 1, \dots, n \tag{3.3}$$

where $N_0 = N_{r+1} = 0$. After the dualization the coefficients \tilde{b} for the beta functions in the internal nodes result

$$\tilde{b}_{2k} = 2N_{2k+1} + 2N_{2k-1} - 3N_{2k} \tag{3.4}$$

$$\tilde{b}_{2k+1} = N_{2k} + N_{2k+2} - N_{2k+1} - 2N_{2k-1} - 2N_{2k+3} \tag{3.5}$$

where k runs from 1 to $\frac{n-1}{2}$, and $N_{n+1} = N_{n+2} = 0$. For the external nodes we have

$$\tilde{b}_1 = N_1 + N_2 - 2N_3 \quad \tilde{b}_n = N_n + N_{n-1} - 2N_{n-2} \tag{3.6}$$

To visualize the resulting magnetic theory (3.2) we exhibit below the content of the magnetic dual theory for an A_5 quiver (see figure 2), which encodes the relevant features.

The superpotential is

$$\begin{aligned} W = & h \left(M_{11}q_{12}q_{21} + M_{13}q_{32}q_{21} + M_{31}q_{12}q_{23} + M_{33}^{(2)}q_{32}q_{23} \right) + \\ & + h \left(M_{33}^{(4)}q_{34}q_{43} + M_{35}q_{54}q_{43} + M_{53}q_{34}q_{45} + M_{55}q_{54}q_{45} \right) + \\ & + hm \left(M_{11}^2 + M_{13}M_{31} + M_{33}^{(2)2} + M_{33}^{(2)}M_{33}^{(4)} + M_{33}^{(4)2} + M_{35}M_{53} + M_{55}^2 \right) + \\ & + h \left(\mu_1^2 M_{11} + \mu_{3,(2)}^2 M_{33}^{(2)} + \mu_{3,(4)}^2 M_{33}^{(4)} + \mu_5^2 M_{55} \right) \end{aligned} \tag{3.7}$$

4. Metastable vacua in A_3 quivers

We start studying the existence and the slow decay of non supersymmetric meta-stable vacua in A_3 quiver gauge theory, the simplest example of an A_n theory. The A_3 gauge group is $U(N_1) \times U(N_2) \times U(N_3)$. As already mentioned in section 2 for a A_n theory, we integrate out the adjoint fields and we perform Seiberg duality on the central node under the constraint

$$N_2 + 1 \leq N_1 + N_3 < \frac{3}{2}N_2 \tag{4.1}$$

The superpotential reads

$$W = h(M_{1,1}q_{1,2}q_{2,1} + M_{1,3}q_{3,2}q_{2,1} + M_{3,1}q_{1,2}q_{2,3} + M_{3,3}q_{3,2}q_{2,3}) + h\mu_1^2 M_{1,1} + h\mu_3^2 M_{3,3} \quad (4.2)$$

where all the mass terms for the mesons have been neglected. Turning on these terms does not ruin the metastability analysis at least for very small masses compared to the supersymmetry breaking scale. Such deformations slightly shift the value of the pseudomoduli in the non supersymmetric minimum, breaking R-symmetry [10]. We neglect them in the following.

The central node yields the magnetic gauge group $U(N_1 + N_3 - N_2)$ whereas the groups at the two external nodes are considered as flavour groups, much less coupled. We discuss in section 5 the consistency of this assumption. Since the gauge group is IR free in the low energy description, and the flavours are less coupled, we are allowed to neglect Kahler corrections and take it as canonical [1]. Moreover the D -term corrections to the one loop effective potential due to the flavour nodes are negligible with respect to the F -term corrections.

Now, there are two different choices of ranks for the A_3 theories, which can give meta-stable vacua: the first possibility is that $N_1 < N_2 \leq N_3$, the second one is $N_1 < N_2 > N_3$. We study separately the two cases which show meta-stable vacua in a similar manner.

$N_1 < N_2 \leq N_3$. We analyze here the case $N_1 < N_2 < N_3$; the equal ranks limit can be easily included. After the dualization the ranks obey the following inequalities $N_1 < \tilde{N}_2 = N_1 + N_3 - N_2 < N_3$.

We work in the regime where $|\mu_1| > |\mu_3|$, and we comment on what happens in the opposite limit in the appendix A, where we shall discuss dangerous tachyonic directions in the quark fields.

We find that the following vacuum is a non supersymmetric tree level minimum

$$q_{1,2} = q_{2,1} = \mu_1 \begin{pmatrix} \mathbf{1}_{N_1} & 0 \end{pmatrix} \quad q_{2,3} = q_{3,2} = \begin{pmatrix} 0 & \mu_3 \mathbf{1}_{\tilde{N}_2 - N_1} \\ 0 & 0 \end{pmatrix} \\ M_{1,1} = 0 \quad M_{1,3} = M_{3,1} = 0 \quad M_{3,3} = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix} \quad (4.3)$$

where the field X is the pseudomodulus, which is a massless field not associated with any broken global symmetries. This flat direction has to be stabilized by the one loop corrections. We start the one loop analysis by rearranging the fields and expanding around the vevs

$$q = \left(\begin{array}{c} q_{1,2} \\ q_{3,2} \end{array} \right) = \left(\begin{array}{cc|cc} \mu_1 + \Sigma_1 & \Sigma_2 & & \\ \Sigma_3 & \mu_3 + \Sigma_4 & & \\ \Phi_1 & & \Phi_2 & \end{array} \right) \quad \tilde{q} = \left(q_{2,1} | q_{2,3} \right) = \left(\begin{array}{c|cc} \mu_1 + \Sigma_5 & \Sigma_6 & \Phi_3 \\ \Sigma_7 & \mu_3 + \Sigma_8 & \Phi_4 \end{array} \right) \\ M = \left(\begin{array}{c|c} M_{1,1} & M_{1,3} \\ \hline M_{3,1} & M_{3,3} \end{array} \right) = \left(\begin{array}{ccc|c} \Sigma_9 & \Sigma_{10} & \Phi_5 & \\ \Sigma_{11} & \Sigma_{13} & \Phi_6 & \\ \Phi_7 & \Phi_8 & X + \Sigma & \end{array} \right) \quad (4.4)$$

We now compute the superpotential at the second order in the fluctuations. We find that the non supersymmetric sector is a set of decoupled O’Raifeartaigh like models with superpotential

$$W = h\mu_3^2 X + hX(\Phi_1\Phi_3 + \Phi_2\Phi_4) + h\mu_3(\Phi_1\Phi_5 + \Phi_2\Phi_6) + h\mu_1(\Phi_3\Phi_7 + \Phi_4\Phi_8) \quad (4.5)$$

In this way all the pseudomoduli can get a mass. The quantum corrections behave exactly as in [1], which means that the pseudomoduli get positive squared mass around the origin of the field space.

The choice (4.3) guarantees that there are no tachyonic directions and have to be made coherently with the hierarchy of the couplings μ_i ; see the appendix A for details.

The lifetime of the non supersymmetric vacuum is related to the value of the scalar potential in the minimum, and to the displacement of the vevs of the fields between the false and the true vacuum. The scalar potential in the non supersymmetric minimum is

$$V_{\min} = (N_3 + N_1 - \tilde{N}_2)|h\mu_3^2|^2 = N_2|h\mu_3^2|^2 \quad (4.6)$$

The vevs of the fields in the supersymmetric vacuum have to be studied considering the non perturbative contributions arising from gaugino condensation. When we take into account these non perturbative effects, we expect that the mesons get large vevs and this allows us to integrate out the quarks using their equation of motion, $q_{i,j} = 0$. In the supersymmetric vacua also $M_{1,3} = 0$ and $M_{3,1} = 0$. If we define

$$M = \begin{pmatrix} M_{1,1} & 0 \\ 0 & M_{3,3} \end{pmatrix} \quad (4.7)$$

the effective superpotential is

$$W = (N_1 + N_3 - N_2) \left(\det(hM) \Lambda_{2i}^{2N_1+2N_3-3N_2} \right)^{\frac{1}{N_1+N_3-N_2}} - h \left(\mu_1^2 \text{tr} M_{1,1} + \mu_3^2 \text{tr} M_{3,3} \right) \quad (4.8)$$

We have now to solve the equation of motion for M_1 and M_3 . The equations to be solved are

$$\begin{aligned} \left(h^M M_{1,1}^{(N_2-N_3)} M_{3,3}^{N_3} \Lambda_{2i}^{(2N_1+2N_3-3N_2)} \right)^{\frac{1}{N_1+N_3-N_2}} - \mu_1^2 &= 0 \\ \left(h^{N_2} M_{1,1}^{N_1} M_{3,3}^{(N_2-N_1)} \Lambda_{2i}^{(2N_1+2N_3-3N_2)} \right)^{\frac{1}{N_1+N_3-N_2}} - \mu_3^2 &= 0 \end{aligned} \quad (4.9)$$

The vevs of the mesons follow solving (4.9)

$$\langle hM_{1,1} \rangle = \mu_1^{\frac{2N_1-N_2}{N_2}} \mu_3^{\frac{2N_3}{N_2}} \Lambda_{2i}^{\frac{3N_2-2N_3-2N_1}{N_2}} \mathbf{1}_{N_1} \quad \langle hM_{3,3} \rangle = \mu_1^{\frac{2N_1}{N_2}} \mu_3^{\frac{2N_3-N_2}{N_2}} \Lambda_{2i}^{\frac{3N_2-2N_3-2N_1}{N_2}} \mathbf{1}_{N_3} \quad (4.10)$$

Since $|\mu_1| > |\mu_3|$, it follows that $\langle hM_{3,3} \rangle > \langle hM_{1,1} \rangle$. This implies that in the evaluation of the bounce action, with the triangular barrier [16], we can consider only the displacement of M_3 in the field space. We obtain for the bounce action

$$S \sim \frac{(\Delta\Phi)^4}{\Delta V} = \left(\frac{\mu_1}{\mu_3} \right)^{\frac{3N_2-2N_3}{N_2}} \left(\frac{\Lambda_{2i}}{\mu_1} \right)^{4\frac{3N_2-2N_3-2N_1}{N_2}} \quad (4.11)$$

Both exponents are positive in the range (4.1). This implies that $S_B \gg 1$, and the vacuum is long living.

$N_1 < N_2 > N_3$. The ranks of the groups after the duality obey the relation $N_1 > \tilde{N}_2 = N_1 + N_3 - N_2 < N_3$. We choose now $|\mu_1| > |\mu_3|$, but we show in the appendix A that also the other choice is possible, leading to other vacua. In the meta-stable vacuum all the vevs of the fields have to be chosen to be zero except a block of the quarks $q_{1,2}$ and $q_{2,1}$ and the pseudomoduli. The vevs are

$$q_{1,2} = \mu_1 \begin{pmatrix} \mathbf{1}_{N_1} \\ \mathbf{0} \end{pmatrix} \quad q_{2,1}^T = \mu_1 \begin{pmatrix} \mathbf{1}_{N_1} \\ \mathbf{0} \end{pmatrix} \quad (4.12)$$

The pseudomoduli come out from the meson $M_{3,3}$ and a $(\tilde{N}_2 - N_1) \times (\tilde{N}_2 - N_1)$ diagonal block of the other meson, $M_{1,1}$. The one loop analysis is the same as before and lifts all the flat directions.

In order to estimate the lifetime we need the vevs of the fields in the supersymmetric vacuum, which are again (4.10), and the value of the scalar potential in the non supersymmetric vacuum (4.12)

$$V_{\min} = (N_2 - N_3)|h\mu_1|^2 + N_3|h\mu_3|^2 \quad (4.13)$$

Since $|\mu_1| > |\mu_3|$ we approximate the scalar potential by the term $\sim |\mu_1|^2$ and the field displacement by $\langle hM_3 \rangle$, obtaining as bounce action

$$S \sim \left(\frac{\mu_1}{\mu_3}\right)^{2\frac{N_2-N_3}{N_2}} \left(\frac{\Lambda_{2i}}{\mu_1}\right)^{4\frac{3N_2-2N_1-2N_3}{N_2}} \gg 1 \quad (4.14)$$

5. Renormalization group flow

The analysis of sections 3 and 4 relies on the fact that we neglect the contributions to the dynamics due to the odd nodes. It means that these groups have to be treated as flavours groups, i.e. global symmetries. However, in the A_n quiver theory each node represents a gauge group factor and we have to analyze how its coupling runs with the energy.

The magnetic window (3.1) constraints the even nodes to be UV free in the high energy description, i.e. $b_{2i} > 0$. The odd groups are not uniquely determined by (3.1) and can be both UV free or IR free in the electric description. In the first case we will choose their scale Λ_{2i+1} to be much lower than the even one

$$\Lambda_{2i+1} \ll \Lambda_{2i}. \quad (5.1)$$

In the second case, when $b_{2i+1} < 0$, Λ_{2i+1} is a Landau pole and we take

$$\Lambda_{2i+1} \gg \Lambda_{2i}. \quad (5.2)$$

In these regimes the even nodes become strongly coupled before the odd ones in the flow toward the infrared. This means that we need a new description provided by Seiberg dualities on the even nodes.

In order to trust the perturbative description at low energy, we have to impose that at the supersymmetry breaking scale (typically μ_i) the odd nodes (flavour), are less coupled than the even ones (gauge), which are always IR free. This requirement will give other constraints on the scales.

As already said there are two possible behaviors of the flavour groups above the scale Λ_{2i} : they can be IR free or UV free. For both cases there are three different possibilities about the beta coefficients in the low energy description.

We start discussing the case when the flavours group are UV free in the electric description. The following three possibilities arise for each flavour group $U(N_{2k+1})$ in the dual theory (Plots 1,2,3 in figure 3).

1. The first one is characterized by

$$b_{2k+1} > 0 \quad \tilde{b}_{2k+1} < \tilde{b}_{2i} < 0 \quad (5.3)$$

In this case the flavour groups $U(N_{2k+1})$ are more IR free than the even nodes after Seiberg duality. The couplings of the flavour groups become more and more smaller than the couplings of the gauge groups along the flow toward low energy. Hence we do not need other constraints on the scales except (5.1).

2. The second possibility is reported in Plot 2 in figure 3

$$b_{2k+1} > 0 \quad \tilde{b}_{2i} < \tilde{b}_{2k+1} < 0 \quad (5.4)$$

The flavour groups $U(N_{2k+1})$ are IR free in the dual theory, but less than the $U(\tilde{N}_{2i})$ gauge groups (5.4). Below a certain energy scale the flavours become more coupled than the gauge groups. If this happens before the supersymmetry breaking scale we cannot trust our description anymore. To solve this problem we have to choose the correct hierarchy between the electric scales of the flavour and the gauge groups, and the supersymmetry breaking scale. We impose that the couplings of the flavours are smaller than the couplings of the gauge groups at the breaking scale, in the magnetic description. This condition can be rewritten in terms of electric scales only using the matching between the magnetic and the electric scales of the flavours. This procedure is explained in the appendix B and gives the following condition on Λ_{2k+1}

$$\Lambda_{2k+1} \ll \left(\frac{\mu}{\Lambda_{2i}} \right)^{\frac{\tilde{b}_{2k+1} - \tilde{b}_{2i}}{b_{2k+1}}} \Lambda_{2i} \ll \Lambda_{2i} \quad (5.5)$$

This imposes a constraint stronger than (5.1) on the strong coupling scale of the flavours.

3. The third possibility (Plot 3 figure 3) is

$$b_{2k+1} > 0 \quad \tilde{b}_{2k+1} > 0 \quad (5.6)$$

In this case the flavour group $U(N_{2k+1})$ is asymptotically free in the low energy description. Once again we have to impose that at the breaking scale the flavours

are less coupled than the gauge groups. The procedure is the same outlined above, and the condition is the same as (5.5). This case may become problematic in the far infrared. Indeed, since the flavour group is UV free, it develops strong dynamics at low energy. If we take into account the non perturbative contributions they could restore supersymmetry. Another interesting feature is the appearance of cascading gauge theories, flowing in the IR. We do not discuss these issues here.

If the flavour groups $U(N_{2k+1})$ are IR free in the electric description the same three possibilities discussed above arise (see Plots 4, 5, and 6 of figure 3).

4. The plot 4 of figure 3 is characterized by

$$b_{2k+1} < 0 \quad \tilde{b}_{2k+1} < \tilde{b}_{2i} < 0 \quad (5.7)$$

Here we do not need any other constraint except (5.2).

5. The plot 5 in figure 3 is

$$b_{2k+1} < 0 \quad \tilde{b}_{2i} < \tilde{b}_{2k+1} < 0 \quad (5.8)$$

The requirement that the odd nodes are less coupled than the even ones at the supersymmetry breaking scale give once again non trivial constraints, with the same procedure outlined previously

$$\Lambda_{2k+1} \gg \left(\frac{\Lambda_{2i}}{\mu} \right)^{\frac{\tilde{b}_{2i} - \tilde{b}_{2k+1}}{b_{2k+1}}} \Lambda_{2i} \gg \Lambda_{2i} \quad (5.9)$$

where now the strong coupling scale of the flavour groups in the electric description is a Landau pole.

6. The last possibility (Plot 6 of figure 3)

$$b_{2k+1} < 0 \quad \tilde{b}_{2k+1} > 0 \quad (5.10)$$

lead to the same constraint (5.9). In the far infrared the strong dynamics of the flavours node can lead to non perturbative phenomena, as in the case 3.

6. Meta-stable A_n

We work in the regime where the ratio $\frac{\mu_i^2}{m}$ is larger than the strong scale of the even nodes Λ_{2i} . This requirement is satisfied if $\lambda_i \gg \Lambda_{2i}^2$ in the electric theory. This allows us to ignore in the dual superpotential (3.2) the presence of quadratic deformations in the mesonic fields.

In this approximation the superpotential of the A_n quiver (3.2) reduces to $\frac{n-1}{2}$ copies of A_3 superpotentials. Hence a generic A_n diagram results decomposable in copies of A_3 quivers, where every adjacent pair shares an odd node.

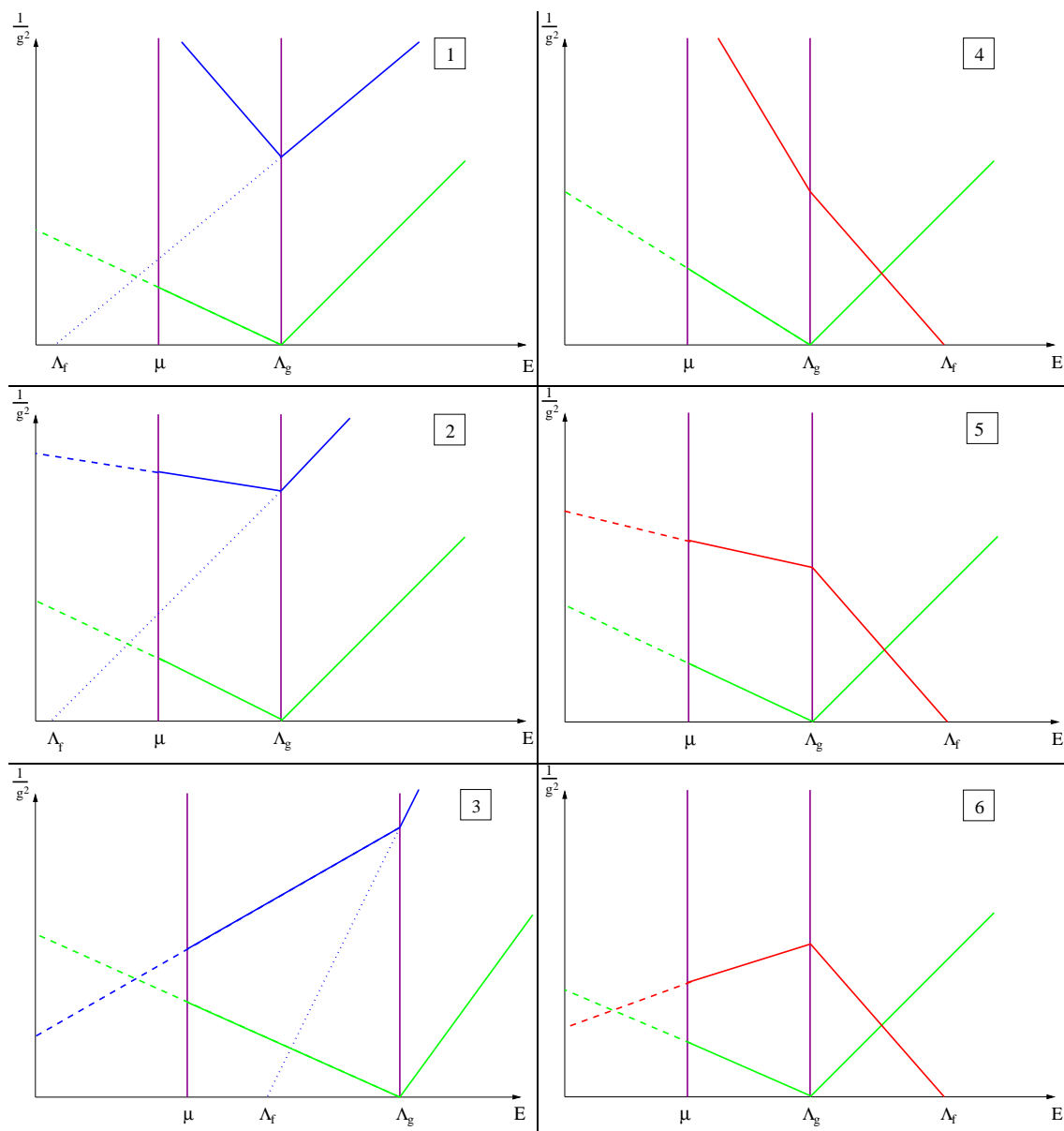


Figure 3: The blue lines refer to flavour/odd groups which are UV free in the electric description, while the red ones are IR free. The green lines refer to the gauge/even group couplings. We denote with μ the supersymmetry breaking scale, and Λ_G and Λ_F are the strong coupling scales of the gauge and the flavour groups, respectively.

For each A_3 the even nodes provide the magnetic gauge groups, and each A_3 has long living metastable vacua, if the perturbative window is correct. It follows that the A_n quiver theory, which is a set of metastable A_3 quivers, possesses metastable vacua.

We still have to be sure of the perturbative regime. This means that we have to control the gauge contributions from the odd nodes of the A_n diagram. We have to proceed as in section 5, and study the beta coefficients of the groups. From (3.5) we can see that the magnetic beta coefficients of the internal odd nodes involve the ranks of the next to next

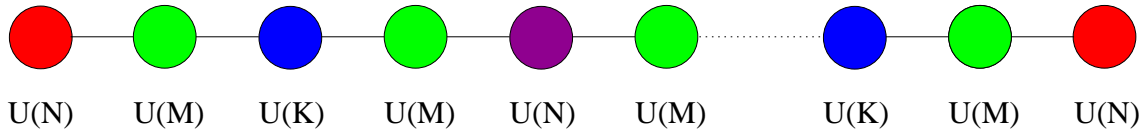


Figure 4: An example with three ranks.

node	b	\tilde{b}
$1, n$ (red)	$3N - M$	$N - 2K + M$
$2i$ (green)	$3M - N - K$	$2K + 2N - 3M$
$4i - 1$ (blue)	$3K - 2M$	$2M - 4N - K \quad i = 1, \dots, \frac{n-1}{4}$
$4j + 1$ (violet)	$3N - 2M$	$2M - 4K - N \quad j = 1, \dots, \frac{n-5}{4}$

Table 2: b and \tilde{b} factors for the example of figure 4.

neighbor groups, i.e. they depend on five integer numbers. This means that in order to know these beta coefficients it is enough to study the A_5 consistent with (3.1). In the appendix C we classify all the possible metastable A_5 diagrams and we give the corresponding electric and magnetic beta coefficients of the central flavour node. This classification describes the RG behaviour of all the internal odd nodes of the A_n .

The running of the first and of the n -th node of the A_n quiver is still undefined and it is discussed in the appendix C.

This provides a classification of metastable A_n quiver gauge theories with alternate Seiberg dualities.

6.1 Example

We show now a simple example of metastable A_n diagram. We choose the even nodes in the electric description to become strongly coupled at the same scale Λ_{2i} . We require that at such scale the flavours (odd nodes) are less coupled than the gauge ones. Moreover we will show that we can also require that in the low energy description all the nodes are IR free and also that the flavour groups (odd nodes) are less coupled than the gauge groups (even nodes) at any scale below the Λ_{2i} .

We study an A_n theory, where $n = 4k + 1$, with k integer. The chain is built as in figure 4, with $N < M < K$. This range allows for metastable vacuum in each A_3 piece as showed previously. We perform alternate Seiberg dualities, working in the in the window

$$M + 1 < N + K < \frac{3}{2}M$$

Thanks to the simple choice for the ranks we have four values for the b coefficients of the beta functions in the electric description, and four values for the coefficients \tilde{b} . They are summarized in table 2.

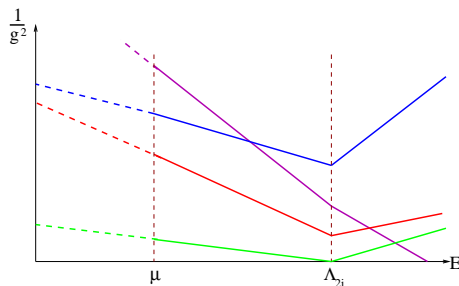


Figure 5: The green line represents the running of the coupling of the even sites. The violet line is related to the $4j + 1$ -th sites, the blue one to the $4i - 1$ -th sites and the red to the first and the last nodes.

We require that in the magnetic description all the nodes are IR free. Moreover we require the beta coefficients of the odd groups to be lower than the even group ones, i.e. $\tilde{b}_{\text{odd}} < \tilde{b}_{2i}$. This restricts the window to

$$K > 2N \qquad 3N < 2M < 4N + K \qquad (6.1)$$

In this regime all the nodes in the electric description are UV free except the $4j + 1$ -th ones. Seiberg duality is allowed on the even nodes, if we impose the following hierarchy of scales

$$\Lambda_1, \Lambda_n, \Lambda_{4i-1} \ll \Lambda_{2i} \ll \Lambda_{4j+1} \qquad (6.2)$$

The running of the gauge couplings of the different nodes are depicted in figure 5.

At high energy the $4j + 1$ -th nodes are strongly coupled, while the other nodes are all UV free. At the scale Λ_{2i} the even nodes become strongly coupled and Seiberg dualities take place. All the runnings of the couplings are changed by these dualities, and all the coefficients of the beta functions \tilde{b}_i become negative. Hence at energy scale lower than Λ_{2i} the theory is weakly coupled. Furthermore the beta coefficients of the odd nodes are more negative than the even node ones. This guarantees that we can rely on perturbative computations, treating the odd nodes as flavours.

7. Gauge mediation

The models analyzed in this work can admit mechanisms of gauge mediation. This means that the breaking of supersymmetry can be transmitted to the Standard Model sector via a gauge interaction. This idea has already appeared in the literature of metastable vacua in A_n theories [8, 9].

Different realizations are possible here. A first one, of direct gauge mediation, identifies the SM gauge group with a subgroup of a flavour group in the quiver [8] and leads to a gaugino mass consistently with the bound of [10].

A second possibility [9] is to connect one of the extremal nodes of the A_n quiver with a new gauge group, which represents the Standard Model gauge group. The arrows connecting these nodes are associated with the messengers f and \tilde{f} , which communicate

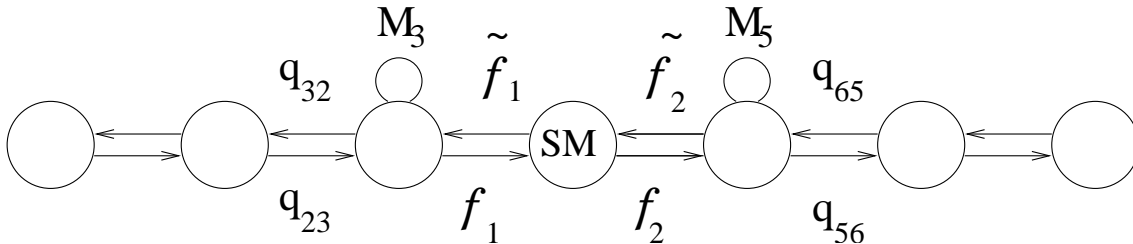


Figure 6: Gauge mediation.

the breaking of supersymmetry to the standard model. Neglecting all the quartic terms, except the term which couples the messengers f, \tilde{f} with the last meson, it is possible to show that also in this case gaugino masses arise at one loop.

In our models of metastable A_n quivers another possibility arises for gauge mediation. It consists in substituting an even node with the Standard Model gauge group (figure 6).

The low energy description is constituted by two metastable A_n (A_3 in this case) which are connected through the SM sector. Both communicate the supersymmetry breaking to the standard model. The superpotential leads to two copies of messengers fields related to the two different hidden sectors

$$W = (m_1 + \theta^2 h_1 F_{M_3}) f_1 \tilde{f}_1 + (m_2 + \theta^2 h_2 F_{M_5}) f_2 \tilde{f}_2 \quad (7.1)$$

A gaugino mass arises at one loop proportional to $\left(h_1 \frac{F_{M_3}}{m_1} + h_2 \frac{F_{M_5}}{m_2} \right)$.

8. Conclusions

We have studied metastability in models of A_n quiver gauge theories. The low energy description in terms of macroscopic fields can be achieved via Seiberg dualities at chosen nodes in the A_n diagram. This choice defines, to a certain extent, the models.

A strategy for building acceptable models unfolds from the request for a reliable perturbative analysis. This constrains the ranks of the gauge groups associated with the nodes and their strong coupling scales. We chose to dualize alternate nodes and we fixed two scales: a unique breaking scale μ and a common strong coupling scale Λ_G for each dualized node. The RG flows of the dualized and non dualized gauge groups must be such that at energy scale higher than μ the gauge groups of the dualized nodes are more coupled than the other ones.

The RG properties of the different nodes of an A_n quiver can be studied decomposing it in A_5 quivers and the decomposition of the A_n in A_3 patches gives the structure of the metastable vacuum. In this way we classify all the possible A_n quiver gauge theories which show metastable vacua with the technique of alternating Seiberg dualities.

Finally we have discussed different patterns of gauge mediation.

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A. Goldstone bosons

The analysis we made in the A_3 theories started from the limit $|\mu_1| > |\mu_3|$. Also the opposite limit can give meta-stable vacua. To understand the differences among the various choices, we have to study the classical masses acquired by the fields expanding them around their vevs.

We study the case with ranks $N_1 < \tilde{N}_2 < N_3$. Since the flavor symmetry is $U(N_1) \times U(N_3)$, and not $U(N_1 + N_3)$, the linear terms of the mesons are different. We are still free to choose the hierarchy between them. We here analyze the breaking of the global symmetries taking $|\mu_1| > |\mu_3|$. Treating the gauge symmetry as a global one, and rearranging the quarks in the form

$$\langle q \rangle = \begin{pmatrix} q_{1,2} \\ q_{3,2} \end{pmatrix} = \begin{pmatrix} \mu_1 \mathbf{1}_{N_1} & 0 \\ 0 & \mu_3 \mathbf{1}_{\tilde{N}_2 - N_1} \\ 0 & 0 \end{pmatrix} \langle \tilde{q}^T \rangle = \begin{pmatrix} q_{2,1} \\ q_{2,3} \end{pmatrix} = \begin{pmatrix} \mu_1 \mathbf{1}_{N_1} & 0 \\ 0 & \mu_3 \mathbf{1}_{\tilde{N}_2 - N_1} \\ 0 & 0 \end{pmatrix} \quad (\text{A.1})$$

we see that the global symmetry breaks as

$$U(N_1) \times U(\tilde{N}_2) \times U(N_3) \longrightarrow U(N_1)_D \times U(\tilde{N}_2 - N_1)_D \times U(N_1 + N_2 - \tilde{N}_2) \quad (\text{A.2})$$

This implies that the Goldstone bosons are $\tilde{N}_2^2 + 2(\tilde{N}_2 - N_1)(N_1 + N_3 - \tilde{N}_2)$. The first \tilde{N}_2^2 Goldstone bosons come from the upper $\tilde{N}_2 \times \tilde{N}_2$ block matrices in the quark fields, exactly the same as in ISS. The second part is a bit different. In fact in ISS, with equal masses, the Goldstone bosons which come from the lower $(N_1 + N_3 - \tilde{N}_2) \times \tilde{N}_2$ sector in the quarks matrices, are $2\tilde{N}_2(N_3 + N_1 - \tilde{N}_2)$. In this case, since we started with lesser flavor symmetry, there are $2N_1(N_3 + N_1 - \tilde{N}_2)$ massless Goldstone bosons fewer than in ISS. We have to control the other directions. From the scalar potential we have to compute the masses that the fields acquire expanding around the vacuum. The relevant expansions for the potentially tachyonic directions are the ones around the vevs of the quarks

$$\begin{aligned} q_{12} &= \begin{pmatrix} \mu_1 + \phi_1 & \phi_2 \end{pmatrix} & q_{21} &= \begin{pmatrix} \mu_1 + \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{pmatrix} \\ q_{23} &= \begin{pmatrix} \phi_3 & \mu_3 + \phi_4 \\ \phi_5 & \phi_6 \end{pmatrix} & q_{32} &= \begin{pmatrix} \tilde{\phi}_3 & \tilde{\phi}_5 \\ \mu_3 + \tilde{\phi}_4 & \tilde{\phi}_6 \end{pmatrix} \end{aligned} \quad (\text{A.3})$$

The relevant terms of the scalar potential come from the F -terms of the mesons

$$V = |F_{M_{11}}|^2 + |F_{M_{13}}|^2 + |F_{M_{31}}|^2 + |F_{M_{33}}|^2 \quad (\text{A.4})$$

If we study the mass terms of the fields ϕ_5 and $\tilde{\phi}_5$ we note that they are not zero, since $\mu_1 \neq \mu_3$. In fact their mass matrix is¹

$$\begin{pmatrix} \phi_5 & \tilde{\phi}_5^\dagger \end{pmatrix} \begin{pmatrix} \mu_1^2 & -\mu_3^2 \\ -\mu_3^2 & \mu_1^2 \end{pmatrix} \begin{pmatrix} \phi_5^\dagger \\ \tilde{\phi}_5 \end{pmatrix} \tag{A.5}$$

with eigenvalues $\mu_1^2 \pm \mu_3^2$. A minimum of the scalar potential without tachyonic directions imposes a constraint on the masses, $\mu_1 > \mu_3$, consistent with the analysis of ISS.

We can ask now what happens if $\mu_1 < \mu_3$. The vacua we studied before are not true vacua any longer, but they have tachyonic directions in the quark fields. The meta-stable vacua are obtained choosing the vevs of $q_{1,2}$ and $q_{2,1}$ to be zero, and the vevs of the other quarks to be

$$q_{3,2} = q_{2,3}^T = \begin{pmatrix} \mu_3 \mathbf{1}_{\tilde{N}_2} \\ 0 \end{pmatrix} \tag{A.6}$$

The differences in the two cases are the value of the scalar potential and the pseudo-moduli. In fact in the first limit $V_{\text{vac}} = (N_1 + N_3 - \tilde{N}_2) |h\mu_3^2|^2$, and in the second limit the scalar potential is $V_{\text{vac}} = (N_3 - \tilde{N}_2) |h\mu_3^2|^2 + N_1 |h\mu_1^2|^2$. Since we choose the masses to be different, but of the same order, both cases have long lived meta-stable vacua. As far as the pseudo-moduli are concerned, in the case analyzed during the paper, they come out from a block of the $M_{3,3}$ meson, and in this case they come out from the whole M_1 meson and from a diagonal block $(N_3 - \tilde{N}_2) \times (N_3 - \tilde{N}_2)$ of the $M_{3,3}$ meson.

B. Hierarchy of scales

One of the main approximation we used to find metastable vacua has been to neglect the fact that the odd nodes are gauge nodes. In order to treat them as flavour groups in the region of interest, it is necessary that their gauge couplings are lower than the couplings of the even nodes. We can treat the odd groups as flavour groups only if this relation holds.

In order to substantiate this idea we have to relate the electric scale of the flavour group to the other scales of the theory. The latter ones are the strong coupling scale of the gauge theories, Λ_{2i} , and the supersymmetry breaking scale μ , which is the value of the linear term in the dual version of the theory.

We must impose the groups related to the flavour/odd nodes to be less coupled than the gauge/even groups in the magnetic region. A similar analysis was performed in [5].

There are six possibilities, shown in figure 3 in section 5. We have already discussed what happens in all these different cases. We will now show how to derive the formulas (5.5) and (5.9).

Let's denote by f all the objects related to the flavour group, and by g all the objects related to the gauge group. We have to distinguish four different cases, all with $\tilde{b}_f > \tilde{b}_g$.²

¹From now on we will consider all the mass terms as real.

²The opposite inequality do not require this analysis, since at low energy the flavours are always less coupled than the gauge.

In fact the flavours can be IR free or UV free in the electric description (i.e. above the scale Λ_{2i}) and also UV free or IR free in the magnetic description.

We start studying a single case, and then we will comment about the others. Let's study the case (2) in figure 3, where the flavours are UV free in the electric and IR free in the magnetic description, i.e. $b_f > 0$ and $\tilde{b}_f < 0$.

We require that after Seiberg duality the gauge coupling g_g is larger than the flavour coupling g_f . More precisely we require that this happens at the supersymmetry breaking scale μ

$$\frac{1}{g_f^2(\mu)} > \frac{1}{g_g^2(\mu)} \quad \Rightarrow \quad \tilde{b}_f \log\left(\frac{\tilde{\Lambda}_f}{\mu}\right) < \tilde{b}_g \log\left(\frac{\tilde{\Lambda}_g}{\mu}\right) \quad (\text{B.1})$$

from which follows

$$\tilde{\Lambda}_f > \left(\frac{\tilde{\Lambda}_g}{\mu}\right)^{\frac{\tilde{b}_g - \tilde{b}_f}{\tilde{b}_f}} \tilde{\Lambda}_g > \tilde{\Lambda}_g \quad (\text{B.2})$$

The scale matching relation coming from Seiberg duality

$$\Lambda_g^{3n_g - n_f} \tilde{\Lambda}_g^{2n_f - 3n_g} = \hat{\Lambda}_g^{n_f} \quad (\text{B.3})$$

fixes $\Lambda_g = \tilde{\Lambda}_g$, if we choose the intermediate scale to be $\hat{\Lambda}_g = \Lambda_g$.

For the flavour scale we observe that, at the scale Λ_g , where we perform Seiberg duality, the coupling in the electric description for the odd node is the same that the coupling of the magnetic description, and this implies

$$g_f = \tilde{g}_f \quad \rightarrow \quad \left(\frac{\Lambda_f}{\Lambda_g}\right)^{b_f} = \left(\frac{\tilde{\Lambda}_f}{\Lambda_g}\right)^{\tilde{b}_f} \quad (\text{B.4})$$

We can now write (B.2) in term of the electric scales (Λ_f and Λ_g) using (B.4), and we obtain

$$\Lambda_f < \mu^{\frac{\tilde{b}_f - \tilde{b}_g}{b_f}} \Lambda_g^{\frac{\tilde{b}_g - \tilde{b}_f + b_f}{b_f}} \quad (\text{B.5})$$

Since the exponent of μ is positive we have

$$\frac{\tilde{b}_f - \tilde{b}_g}{b_f} > 0 \quad \rightarrow \quad \Lambda_f < \left(\frac{\mu}{\Lambda_g}\right)^{\frac{\tilde{b}_f - \tilde{b}_g}{b_f}} \Lambda_g \ll \Lambda_g \quad (\text{B.6})$$

This imposes a stronger constraint on the scale of the flavour group Λ_f . In fact it is not enough to choose it lower than the gauge strong coupling scale Λ_g . It is also constrained by (B.6). Figure 7 explains what happens.

In the first picture the scale Λ_f is lower than Λ_g but not enough: at the breaking scale it is not possible to neglect the contribution coming from \tilde{g}_f . Instead, if we constrain the scale Λ_f using (B.6), we obtain the runnings depicted in the second picture: here the flavour groups are less coupled than the gauge groups at the supersymmetry breaking scale.

As explained above there are four different possibilities. The second possibility is that the flavours are UV free both in the electric description and in the magnetic description,

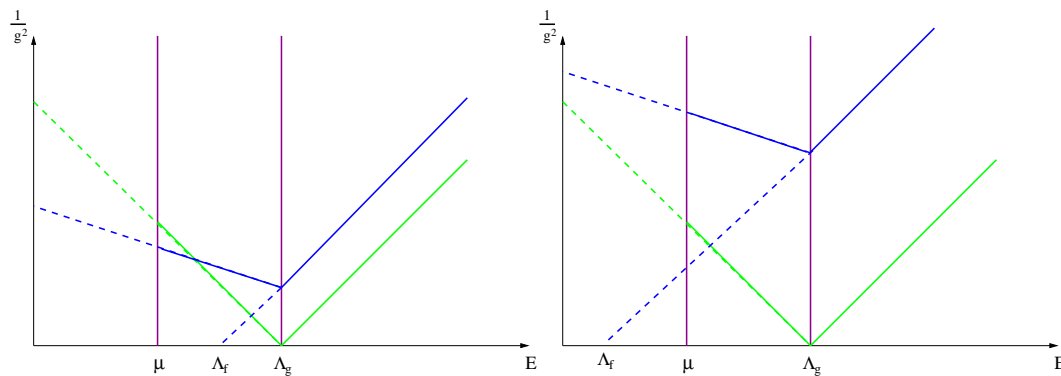


Figure 7: Hierarchy of scales.

with $\tilde{b}_f > 0$. The analysis is the same as before, and we obtain the same inequality as (B.6). However this situation requires a more careful analysis, since in the infrared the gauge coupling associated to the flavour group develops a strong dynamics which has to be taken under control.

For the other two possibilities, where $b_f < 0$, one finds

$$\Lambda_f > \left(\frac{\Lambda_g}{\mu}\right)^{\frac{\tilde{b}_g - \tilde{b}_f}{b_f}} \quad \Lambda_g \gg \Lambda_g \quad (\text{B.7})$$

The general recipe we learn from this analysis can be summarized in three different cases

- If the inequality $\tilde{b}_f < \tilde{b}_g$ holds one has simply to choose $\Lambda_f \ll \Lambda_g$ or $\Lambda_f \gg \Lambda_g$ if $b_f > 0$ or $b_f < 0$ respectively as in (5.1), (5.2).
- If $\tilde{b}_f > \tilde{b}_g$ we can still distinguish two cases
 - In the first case $b_f > 0$, and we have to constraint Λ_f with (B.6).
 - In the second case $b_f < 0$, and we have to constraint Λ_f with (B.7).

C. A_5 classification

We study A_5 quiver gauge theories obtained gluing all the possible combinations of A_3 which present metastable vacua, i.e. the one of section 4.

We analyze the beta function coefficients for these A_5 quiver gauge theories, with gauge group $U(N_1) \times U(N_2) \times U(N_3) \times U(N_4) \times U(N_5)$. The even nodes are in the IR free window

$$N_2 < N_1 + N_3 < \frac{3}{2}N_2 \quad N_4 < N_3 + N_5 < \frac{3}{2}N_4 \quad (\text{C.1})$$

We write in table 3 the beta coefficients of the third node of the A_5 , specifying the range, compatible with (C.1), when this node is UV free or IR free in the electric and in the magnetic descriptions, respectively. The table classifies the possible A_5 quiver gauge theories which present alternate Seiberg dualities and which have metastable vacua.

As explained in section 6 we can obtain an A_n quiver gauge theory by gluing the A_3 patches. For the renormalization group, the internal flavour nodes of the A_n chain behave as the third node of the A_5 patches.

The table does not say anything about the external nodes of the A_n . In the electric theory one has $b_1 = 3N_1 - N_2$ and $b_n = 3N_n - N_{n-1}$; after duality, in the low energy description we have $\tilde{b}_1 = N_1 + N_2 - N_3$, and $\tilde{b}_n = N_n + N_{n-1} - 2N_{n-2}$. The possible values for \tilde{b}_1 and \tilde{b}_n have to be studied separately.

Ranks of A_5	Further condition(I)	Further condition(II)	electric b - factor	magnetic b - factor
$N_1 < N_2 \leq N_3 < N_4 \leq N_5$		$N_2 + N_4 < 3N_3$	$b_3 > 0$	$\tilde{b}_3 < 0$
		$3N_3 < N_2 + N_4$	$b_3 < 0$	$\tilde{b}_3 < 0$
$N_1 < N_2 > N_3 < N_4 \leq N_5$			$b_3 < 0$	$\tilde{b}_3 < 0$
$N_1 \geq N_2 > N_3 < N_4 \leq N_5$			$b_3 < 0$	$\tilde{b}_3 < 0$
$N_1 < N_2 > N_3 < N_4 > N_5$	$N_3 < N_1 + N_5$ $N_3 > N_1 + N_5$	$N_2 + N_4 < 3N_3$	$b_3 > 0$	$\tilde{b}_3 < 0$
		$\frac{3N_3 < N_2 + N_4}{N_2 + N_4 < N_3 + 2N_1 + 2N_5}$	$\frac{b_3 < 0}{b_3 > 0}$	$\frac{\tilde{b}_3 < 0}{\tilde{b}_3 < 0}$
		$N_3 + 2N_1 + 2N_5 < N_2 + N_4$	$b_3 > 0$	$\tilde{b}_3 > 0$
$N_1 < N_2 \leq N_3 \geq N_4 > N_5$		$N_2 + N_4 < N_3 + 2N_1 + 2N_5$	$b_3 > 0$	$\tilde{b}_3 < 0$
		$N_3 + 2N_1 + 2N_5 < N_2 + N_4$	$b_3 > 0$	$\tilde{b}_3 > 0$
$N_1 < N_2 \leq N_3 < N_4 > N_5$	$N_3 < N_1 + N_5$ $N_3 > N_1 + N_5$	$N_2 + N_4 < 3N_3$	$b_3 > 0$	$\tilde{b}_3 < 0$
		$\frac{3N_3 < N_2 + N_4}{N_2 + N_4 < N_3 + 2N_1 + 2N_5}$	$\frac{b_3 < 0}{b_3 > 0}$	$\frac{\tilde{b}_3 < 0}{\tilde{b}_3 < 0}$
		$N_3 + 2N_1 + 2N_5 < N_2 + N_4$	$b_3 > 0$	$\tilde{b}_3 > 0$

Table 3: In the first column we report all the possible inequalities among the A_5 rank numbers consistent with (C.1). Moving from left to right the further condition fix the signs of b_3, \tilde{b}_3 .

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